Principio del Buon Ordinamento

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Sia $n_0 \in \mathbb{N}$ sia un insieme $\mathbb{Z}_{n_0} = \{z \in \mathbb{Z} | z \geq n_0\}$. Allora $\forall X \subseteq \mathbb{Z}_{n_0}$ ammette minimo.

1 Esempio

$\mathbb{Z}_{-5}$ è l'insieme di tutti i numeri razionali maggiori di -5 allora -4 è il minimo.

2 Appunti + Esercizi induzione

- RSA unica domanda teorica

Principio del buon ordinamento (PBO)

$n_\emptyset$ intero

$Z_{n_\emptyset} = \{ z \in Z \mid z \geq n_\emptyset \}$

$\forall X \subseteq Z_{n_\emptyset}$ ammette minimo $x_\emptyset$

$PBO \equiv$ Principio di induzione

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somma di n int $> \emptyset$ è: $\frac{n(n+1)}{2} = \sum_{k=1}^{n} k$

1) $n = 1$ $\quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \Rightarrow \sum_{k=1}^{1} k = \frac{1(1+1)}{2} \Rightarrow 1 = 1$

2) $n = n-1$ $\quad \sum_{k=1}^{n-1} k = \frac{(n-1)(n+1-1)}{2}$

$$n + \sum_{k=1}^{n-1} k = n + \frac{(n-1)n}{2}$$

$$\sum_{k=1}^{n} k = \frac{2n + (n-1)n}{2}$$

$$= \frac{n(2+(n-1))}{2}$$

$$= \frac{n(n+1)}{2}$$

$|P(X)| = 2^n$ numero elementi

insieme delle parti

1) $X = \emptyset$ $\quad |P(\emptyset)| = 2^0 \Rightarrow 1 = 1$

2) $|P(Y)| = |P(X)| - 1$ $\quad |P(Y)| = 2^{n-1}$ è vero

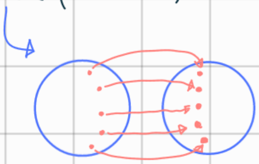
3) $|P(X)| = 2^n$

$x_\emptyset \in X$

$P(X) = C \cup C'$ dove $C = \{ Y \subset X \mid x_\emptyset \notin Y \} = P(X \setminus \{x_\emptyset\})$

$C' = \{ Y \subseteq X \mid x_0 \in Y \} = P(\overline{X \setminus \{x_0\}})$

$C \cap C' = \emptyset$ posso definire una funzione biettiva $(|C| = |C'|)$



$|C| = 2^{n-1}$ quindi $|C'| = 2^{n-1}$

$|C| \cup |C'| = 2^{n-1} + 2^{n-1}$

$P(X) = 2^{n-1}(1+1)$

$= 2^{n-1}(2)$

$= 2^n$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6} n (n+1)(2n+1) \ ?$$

$(2n^2+3n+1)$

BC

$n = 1$

$$\sum_{k=1}^{1} k^2 = 1\frac{1}{6}(1+1)(2+1)$$

$1 = \frac{1}{3} 3$

$1 = 1$ ✓

II

$n = n-1$

$$\sum_{k=1}^{n-1} k^2 = \frac{n-1}{6}(n-1+1)(2(n-1)+1)$$

assumo vero

PI

$2n^2 - n$

$$\left(\sum_{k=1}^{n} k^2 = \right) n^2 + \sum_{k=1}^{n-1} k^2 = \frac{n-1}{6}(n-1+1)(2(n-1)+1) + n^2$$

$$\sum_{k=1}^{n} k^2 = \frac{2n^3 - \cancel{n^2} - \cancel{2n^2} + n + \cancel{6}^{3} n^2}{6}$$

$$\sum_{k=1}^{n} k^2 = \frac{2n^3 + 3n^2 + n}{6}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(2n^2+3n+1)}{6}$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6} n (n+1)(2n+1) \quad \checkmark$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{1}{2} n (n+1)\right)^2 \ ?$$

BC $\quad n = 1$

$$\sum_{k=1}^{1} k^3 = \left(\frac{1}{2}(1+1)\right)^2$$

$1 = 1$ ✓

II

$$\sum_{k=1}^{n-1} k^3 = \left(\frac{1}{2}(n-1)(n-1+1)\right)^2$$

PI

$$n^3 + \sum_{k=1}^{n-1} k^3 = \left(\frac{1}{2}(n-1)(n-1+1)\right)^2 + n^3$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{1}{2}(n-1)(n\cancel{-1+1})\right)^2 + n^3$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n^2-n}{2}\right)^2 + n^3$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4 \cancel{-2n^3} + n^2 + \overset{2}{\cancel{4}}n^3}{4}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4+2n^3+n^2}{4}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n^2+n}{2}\right)^2$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{1}{2}n(n+1)\right)^2$$