

Grammatica

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Una grammatica G è una quadrupla $G = (V, T, Prod, S)$ dove:

- V è l'insieme delle variabili non-terminali
- T è l'insieme dei simboli terminali
- $Prod$ è l'insieme delle regole di produzione
- S è una variabile detta *start symbol*

1 Esercizi

$$L = \{ 0^n 1^n \mid n \geq 1 \} \text{ fornire un CFG}$$

$$L = \{ 01, 0011, 000111, \dots \}$$

$$S \rightarrow 01 \mid 0S1$$

$$G = \{ \{ S \}, \{ 0, 1 \}, \{ S \rightarrow 01 \mid 0S1 \}, S \}$$

$$\text{Fornire CFG per } L = \{ w \in \{ a, b, c, d \}^* \mid w = a^n b^n c^k d^k, \text{ con } n, k \geq 0 \}$$

$$S \rightarrow Z X$$

$$Z \rightarrow \epsilon \mid a Z b$$

$$X \rightarrow \epsilon \mid c X d$$

$$\text{Fornire CFG per } L = \{ a^n c b^n \mid n \geq 1 \}$$

$$S \rightarrow a c b \mid a S b$$

$$\left(\begin{array}{l} \text{con } n \geq 0 \\ S \rightarrow c \mid a S b \end{array} \right)$$

$$\text{Fornire CFG per } L = \{ a^n c^k b^n \mid n, k \geq 0 \}$$

$$S \rightarrow a S b \mid a X b$$

$$X \rightarrow c \mid X c$$

$$\text{CFG per } L = \{ a^{n+m} x c^n w d^m \}$$

$$= \{ a^m a^n x c^n y d^m \}$$

$$S \rightarrow a S d \mid X y$$

$$X \rightarrow x \mid a X c$$

CFG per $L = \{ a^n c b^m c^m a d^m \} \quad n \geq 0, m \geq 1$

$$S \rightarrow X Y$$

$$X \rightarrow c \mid a X b$$

$$Y \rightarrow c a d \mid c X d$$

CFG per $L = \{ a^n b^m \} \quad n \geq m \geq 0$

Se $n \geq m$ allora $\exists k \mid n = m + k$ quindi $L = \{ a^{m+k} b^m \} = \left\{ \begin{array}{l} \{ a^k a^m b^m \} \\ \{ a^m a^k b^m \} \end{array} \right\}$

$$S \rightarrow a S \mid X$$

$$X \rightarrow a X b \mid \varepsilon$$

oppure

$$S \rightarrow a S b \mid X$$

$$X \rightarrow a X \mid \varepsilon$$

CFG per $L = \{ a^n b^{m+n} c^h \} \quad m > h \geq 0, n \geq 0$

$$= \{ a^n b^n b^m c^h \} \quad \exists k \mid m-1 = h+k$$

$$= \{ a^n b^n b^{h+k+1} c^h \}$$

$$= \{ a^n b^n b^h b^k c^h \}$$

$$S \rightarrow X Y$$

$$X \rightarrow \varepsilon \mid a X b$$

$$Y \rightarrow b \mid b Y \mid b Y c$$

$$b \Rightarrow X Y \Rightarrow Y \Rightarrow b$$

$$bb \Rightarrow X Y \Rightarrow Y \Rightarrow b Y \Rightarrow bb$$

$$bbbb \Rightarrow X Y \Rightarrow Y \Rightarrow b Y \Rightarrow bb Y \Rightarrow bbb Y \Rightarrow bbbb \quad \checkmark$$

$$b b b b c \Rightarrow XY \Rightarrow Y \Rightarrow b Y \Rightarrow b b Y \Rightarrow b b b Y c \Rightarrow b b b b c \quad \checkmark$$

$$b c \Rightarrow XY \Rightarrow Y \Rightarrow b Y c \Rightarrow b b c \quad \times$$

$$\begin{aligned} \text{CFG per } L &= \{ a^n b^m c^k \mid \text{con } k > n+m \\ &= \{ a^n b^m c^{n+m+h} \mid h > 0 \mid k = n+m+h \\ &= \{ \underbrace{a^n b^m c^m c^n}_{Y} c^h \} \\ &\quad \underbrace{\hspace{10em}}_X \end{aligned}$$

$$S \rightarrow X c | c$$

$$X \rightarrow \epsilon | a Y c$$

$$Y \rightarrow \epsilon | b Y c$$

$$\begin{aligned} \text{CFG per } L &= \{ a^n x c^{n+m} y^h z^k d^{m+h} \mid n, m, h, k \geq 0 \} \\ &= \{ \underbrace{a^n x c^n}_A \underbrace{c^m y^h z^k d^h}_{B} d^m \} \\ &\quad \underbrace{\hspace{10em}}_C \end{aligned}$$

$$S \rightarrow AB$$

$$A \rightarrow x | a A c$$

$$B \rightarrow C | c B d$$

$$C \rightarrow D | y C d$$

$$D \rightarrow \epsilon | z D$$

