Determinante

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Il determinante è una funzione:

$$\det_n : \{\text{matrici quadrate di ordine } n\} \to \mathbb{R}$$

1 Proprietà caratterizzanti

1. $\det(\underline{c_1}, \cdots, \underline{a+b}, \cdots, \underline{c_n}) = \det(\underline{c_1}, \cdots, a, \cdots, \underline{c_n}) + \det(\underline{c_1}, \cdots, b, \cdots, \underline{c_n})$
2. $\det(\underline{c_1}, \cdots, \lambda * \underline{c}, \cdots, \underline{c_n}) = \lambda * \det(\underline{c_1}, \cdots, \underline{c}, \cdots, \underline{c_n})$
3. $\det(\underline{c_1}, \cdots, \underline{c_i}, \underline{c_{i+1}}, \cdots, \underline{c_n}) = 0$ con $\underline{c_i} = \underline{c_{i+1}}$
4. $\det(\underline{e_1}, \underline{e_2}, \cdots, \underline{e_n}) = 1$ dove e_i è la base canonica di $\mathbb{R}^n$

Le proprietà 1 e 2 sono dette di **multilinearità** mentre la proprietà 3 è detta di **alternanza**.

1.1 Osservazione

La propritià 3 è verificata sse: $\det(\underline{c_1}, \cdots, \underline{c_i}, \underline{c_{i+1}}, \cdots, \underline{c_n}) = -\det(\underline{c_1}, \cdots, \underline{c_{i+1}}, \underline{c_i}, \cdots, \underline{c_n})$. Inoltre $\det(\underline{c_1}, \cdots, \underline{c_i}, \cdots, \underline{c_j}, \cdots, \underline{c_n}) = -\det(\underline{c_1}, \cdots, \underline{c_j}, \cdots, \underline{c_i}, \cdots, \underline{c_n})$.

Par calcolare il determinante di matrici di ordine superiore al secondo si utilizza la formula di Laaplace.

2 Teorema

Esiste un'unica funzione che soddisfi le proprietà caratterizzanti.

3 Esempi

$\det_1 = Id$

$$
\begin{aligned}
&\det_2\left(\begin{bmatrix}a\\c\end{bmatrix}, \begin{bmatrix}b\\d\end{bmatrix}\right) = \\
&\det_2\left(a\begin{bmatrix}1\\0\end{bmatrix} + c\begin{bmatrix}0\\1\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}\right) = \\
&\det_2\left(a\begin{bmatrix}1\\0\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}\right) + \det_2\left(c\begin{bmatrix}0\\1\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}\right) = \\
&a * \det_2\left(\begin{bmatrix}1\\0\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}\right) + c * \det_2\left(\begin{bmatrix}0\\1\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}\right) = \\
&a*\Bigg(\det(\begin{bmatrix}1\\0\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix}) + \det(\begin{bmatrix}1\\0\end{bmatrix}, d\begin{bmatrix}0\\1\end{bmatrix})) + c * (\det(\begin{bmatrix}0\\1\end{bmatrix}, b\begin{bmatrix}1\\0\end{bmatrix}) + \det(\begin{bmatrix}0\\1\end{bmatrix}, d\begin{bmatrix}0\\1\end{bmatrix})) = \\
&a*(b*\det(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}) + d*\det(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix})) + c*(\det(b*\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}) + \\
&d * \det(\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix})) = \\
&a*(b*\det(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}) + d*\det(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix})) + c*(\det(b*(-\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix})) + \\
&d * \det(\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix})) = \\
&a * (b * 0 + d * 1) + c * (b * (-1) + d * 0) = a * d - c * b \\
&\det_2\left(\begin{bmatrix}a\\c\end{bmatrix}, \begin{bmatrix}b\\d\end{bmatrix}\right) = ad - cb
\end{aligned}
$$